A conceptual scheme to generate code from DEB assumptions

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I. Motivation

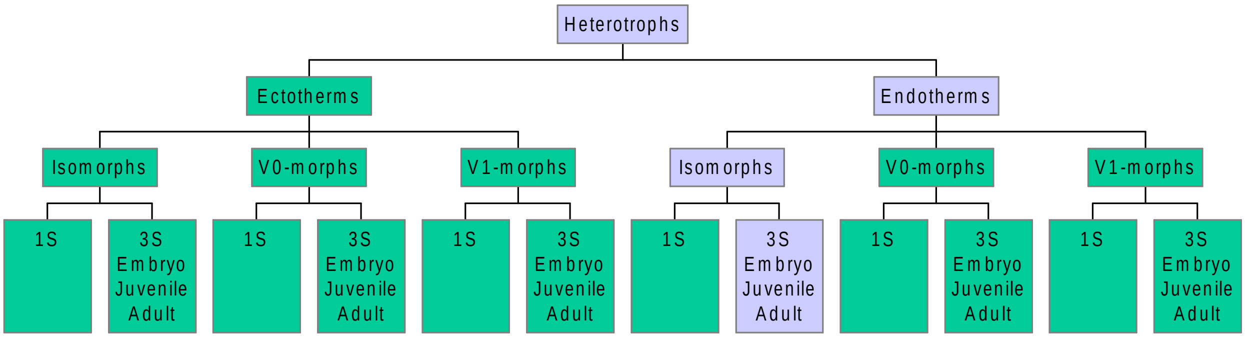
Developing a first approximation to a simple conceptual scheme that would allow an automatic generation of a source code from the assumptions that a user would make in implementing his DEB model. This approach may help the biologist understand the equations and have a computational tool readily at hand.

II. Conceptual Scheme

This scheme is based on a decision tree, which take in account the basis processes that constitute the DEB model. Following the aim of DEB theory, we propose a scheme for "heterotrophs one structure and one reserve" that allow a differentiation of individuals based in their energetic dynamic processes.

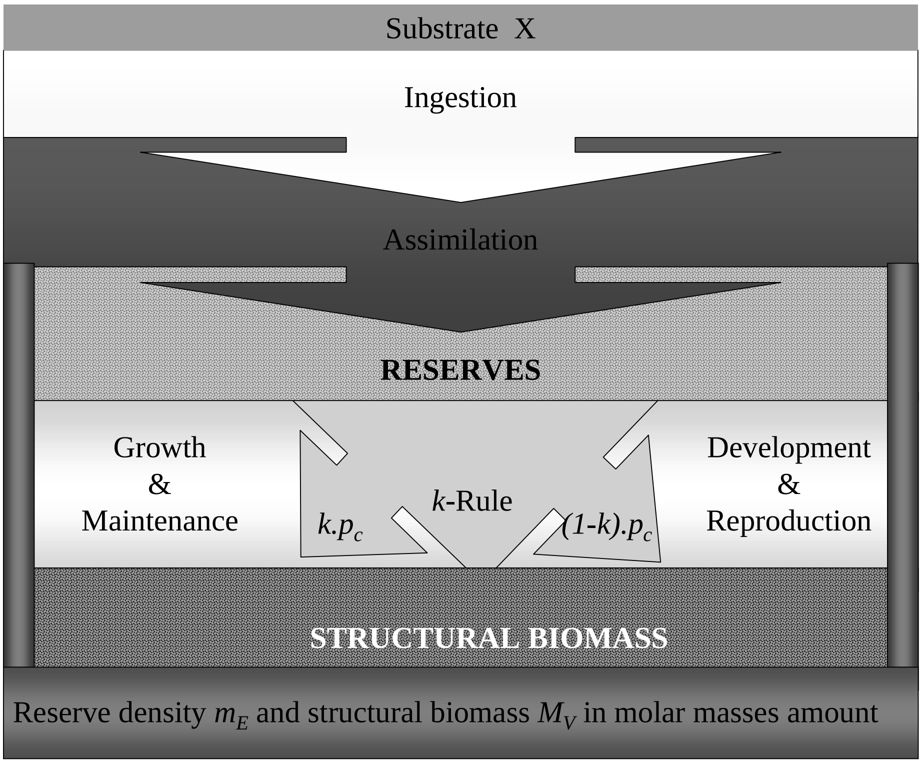
A generation code tool should travel the tree asking to the user the condition of the individual at different levels. Based in the response of the user this tool chooses adequate equation for each power. At the end of the tree all the powers are defined and the general equations are built. Could be important ask for the parameter values of the particular species that the user wants to model to obtain directly a routine ready to run.

The light violet boxes in the tree indicate a path that is fully specified in **Following an Individual**.



III. From Basic Processes to General Equations

As Chapter 3 notes, the relationship between the different processes is chosen following the fate of food.



This is a very simple resume of the different processes, which together constitute de Dynamic Energy Budget model. The description and considerations about this processes are mainly explained in Chapter 3 (Kooijman, 2000).

Ingestion Process

<u>For isomorph</u>

Ingestion rate $\dot{J}_X = \{\dot{J}_{X_m}\} f V^{2/3}$

Proportional to surface area, where f is the functional response and $\{\dot{J}_{X_m}\}$ is the surface-area-specific max ingestion rate.

<u>For no-isomorph</u>

It is necessary to multiply the expression above by the shape correction function, $\left(\frac{V}{V_d}\right)^{-2/3}$ for V_0-morph and $\left(\frac{V}{V_d}\right)^{1/3}$ for V_1-morph.

Assimilation Process

“Assimilated energy”: denotes the free energy fixed into reserves.
The assimilation rate or assimilation power

$$\dot{p}_A[et^{-1}] = \underbrace{\frac{\{\dot{p}_{Am}\}}{\{\dot{J}_{Xm}\}}}_{\substack{\text{conversion efficiency}\\ \text{of food}}} \overbrace{f.\{\dot{J}_{Xm}\}V^{2/3}}^{\text{ingestion rate for iso}}$$

For no-isomorph the ingestion rate is the modified one by the corresponding shape correction function.

The DEB model is built on two state variables, ***Reserves*** ([E]) and ***Structural Biomass*** (V).

Reserves

This is the first approximation to reserve’s dynamics:

$$\frac{d}{dt}E = \dot{p}_A - \underbrace{\dot{p}_c}_{\substack{\text{energy consumed}\\ \text{bybodily tissues}}}$$

Working with the reserve density $[E] = E/V$ eq. 3.7 is obtained:

$$\frac{1}{V}\frac{d}{dt}E = (\dot{p}_A - \dot{p}_c)\ \frac{1}{V}$$

$$\frac{d}{dt}[E] + [E]\frac{d}{dt}\ln V = [\dot{p}_A] - [\dot{p}_C]$$

$$\frac{d}{dt}[E] = [\dot{p}_A] - [\dot{p}_C] - \underbrace{[E]\frac{d}{dt}\ln V}_{\text{diluton by growth}} \quad (3.7)$$

Because maintenance (work) and growth are among the destinations of catabolic energy and k-rule this equation is stated, a fraction k of the is allocated to maintenance and growth:

$$k\ \dot{p}_c = \dot{p}_m + [E_G]\frac{d}{dt}V$$

$$\frac{d}{dt}V = \frac{k\ \dot{p}_c - \dot{p}_m}{[E_G]}$$

$$\frac{d}{dt}\ln V = \frac{1}{V}\frac{d}{dt}V = \frac{1}{V}\frac{k\ \dot{p}_c - \dot{p}_m}{[E_G]} = \frac{k[\dot{p}_c]}{[E_G]} - \frac{[\dot{p}_m]}{[E_G]}$$

From these equations the following expression for [E] is reached:

$$\frac{d}{dt}[E] = \frac{\{\dot{p}_{Am}\}}{V^{1/3}}\left(f - \frac{[E]}{[E_m]}\right) \quad (3.10)$$

Structural Volume

To deduce the equation that describes the dynamic of this variable it is necessary to consider again the fraction of the catabolic power allocated to maintenance and growth.

$$k\ \dot{p}_c = \dot{p}_m + [E_G]\frac{d}{dt}V + \underbrace{\dot{p}_T}_{\text{is 0 for ectotherms}} \qquad (3.17)$$

Where

$$\dot{p}_m = [\dot{p}_m]V \qquad (3.15)$$

$$\dot{p}_T = \{\dot{p}_T\}V^{2/3} \qquad (3.16)$$

$$\dot{p}_c = \dot{p}_A - V\frac{d}{dt}[E] - [E]\frac{d}{dt}V = [E]\left(\dot{v}V^{2/3} - \frac{d}{dt}V\right) \quad \text{from } 3.7$$

Substituting the last expressions in (3.17) an expression for the structural volume is deduced:

$$\frac{d}{dt}V = \dot{v}\,\frac{V^{2/3}\,[E]/[E_m] - V^{2/3}\left(V_h/V_m\right)^{1/3} - V/V_m^{1/3}}{[E]/[E_m] + g} \qquad (3.18)$$

The equations 3.10 and 3.18 lead the dynamic of the system in basis on *specific assumptions* (Table 3.3), which serve to specify the basic powers.
These equations are equivalent to 3.55 and 3.56, which are based on *general assumptions* and take in account the mass-energy relationship.

We provide as an example the deduction of the equivalence of equations 3.10 and 3.55. Mathematically, the demonstration can be done beginning from 3.10 or 3.55 indistinctly.

$$\frac{d}{dt} m_E = j_{EAm}\left(f - \frac{m_E}{m_{Em}} \right) \qquad (3.55)$$

$$\frac{d}{dt}\left(\frac{\frac{E}{\mu_E}}{V[M_V]} \right) = y_{EX}\ j_{XAm}\left(f - \frac{\frac{e[E_m]}{\mu_E [M_V]}}{\frac{[M_{Em}]}{[M_V]}} \right)$$

$$\frac{1}{\mu_E}\frac{d}{dt}\left(\frac{[E]}{[M_V]} \right) = y_{EX}\ \frac{\dot{J}_{XAm}}{M_V}\left(f - \frac{\frac{[E]}{[E_m]}[E_m]}{\mu_E [M_{Em}]} \right)$$

$$\frac{1}{\mu_E}\left(\frac{[M_V]\frac{d}{dt}[E] - [E]\ \overbrace{\frac{d}{dt}[M_V]}^{\text{equal } 0,\, M_V \text{ constant}}}{[M_V]^2} \right) = y_{EX}\ \frac{\{\dot{J}_{XAm}\}V^{2/3}}{[M_V]V}\left(f - \frac{[E]}{\mu_E \frac{[E_m]}{\mu_E}} \right)$$

$$\frac{1}{\mu_E [M_V]^2}\left([M_V]\frac{d}{dt}[E] \right) = \frac{1}{y_{XE}}\ \frac{\{\dot{J}_{XAm}\}}{[M_V]V^{1/3}}\left(f - \frac{[E]}{[E_m]} \right)$$

$$\frac{1}{\mu_E [M_V]}\left(\frac{d}{dt}[E] \right) = \frac{\mu_{AX}}{\mu_E}\ \{\dot{J}_{XAm}\}\ \frac{1}{[M_V]V^{1/3}}\left(f - \frac{[E]}{[E_m]} \right)$$

$$\frac{1}{\mu_E [M_V]}\left(\frac{d}{dt}[E] \right) = \frac{1}{\mu_E [M_V]}\{\dot{p}_{Am}\}\frac{1}{V^{1/3}}\left(f - \frac{[E]}{[E_m]} \right)$$

$$\frac{d}{dt}[E] = \{\dot{p}_{Am}\}\frac{1}{V^{1/3}}\left(f - \frac{[E]}{[E_m]} \right)$$

IV. Following One Individual: the Capybara as an example

We present here the equations as a result of a specific case. We take the equations developed in point II, traveling through the decision tree and generating the related scaled equations.

The capybara, the largest living rodent, has considerable potential as a new meat and hide producing species, which is adapted to seasonally wet flat lands in extensive regions throughout South America. This model is embedded into the INCO DC Project "The sustainable management of wetland resources in Mercosur" (http://www.unisi.it/wetland).

The Capybara (*Hydrochoerus hydrochaeris*) is a heterotroph, multicellular and isomorphic animal that develops through an embryonic, juvenile and adult phase. No assimilation occurs during the embryonic phase. Reproduction allocation occurs during the juvenile phase. The endothermic heating occurs during the juvenile and adult phase.

Currently we are developing an *aggregated model* of this species. An aggregated model is a model that assumes that all individual members of the population can be aggregated into a single state variable representing population size (DeAngelis, D. & Gross, L.,1992). Many classical models in ecology, such as the logistic equation and the Lotka-Volterra equations, assume that all the individuals in a population are identical and can be lumped together. The disappointing performance of aggregated models has led to search for models has led to a search for models of populations and larger-scale ecological systems based upon the characteristics of individuals. But to build a realistic individual based model, a detailed study of the species is needed in order to estimate the parameters of the model. At this moment, in the particular case of capybara, we do not have the necessary field studies to build an individual model.

The general equations

$$\frac{d}{dt}[E] = \frac{\{p_{Am}\}}{V^{1/3}}\left(f - \frac{[E]}{[E_m]}\right) \quad (3.10)$$

$$\frac{d}{dt}V = \dot{v}\,\frac{V^{2/3}[E]/[E_m] - V^{2/3}\left(V_h/V_m\right)^{1/3} - V/V_m^{1/3}}{[E]/[E_m] + g} \quad (3.18)$$

are scaled with $e = \frac{[E]}{[E_m]}$ and $l = \left(\frac{V}{V_m}\right)^{1/3}$, using

(a) $\upsilon = \frac{\{p_{Am}\}}{[E_m]}$ (b) $K_M = \frac{[p_M]}{[E_G]}$ (c) $g = \frac{[E_G]}{\kappa[E_m]}$

(d) $V_m = \left(\frac{\upsilon}{gK_M}\right)^3 = \left(\frac{\kappa\{p_{Am}\}}{[p_M]}\right)^3$ (e) $V_h = \left(\frac{\{p_T\}}{[p_M]}\right)^3$ (f) $l_h = \left(\frac{V_h}{V_m}\right)^{1/3}$

Differentiating $e = \frac{[E]}{[E_m]}$, we have $\frac{de}{dt} = \frac{1}{[E_m]}\frac{d[E]}{dt}$. Substituting 3.10

$\frac{de}{dt} = \frac{1}{[E_m]}\frac{\{p_{Am}\}}{V^{1/3}}\left(f - \frac{[E]}{[E_m]}\right)$ we obtain:

$$\frac{de}{dt} = \frac{\upsilon}{V^{1/3}}(f - e) \quad (3.11)$$

Differentiating $l = \left(\frac{V}{V_m}\right)^{1/3}$, we have $\frac{dl}{dt} = \frac{V^{-2/3}}{3V_m^{1/3}}\frac{dV}{dt}$. Substituting 3.18

$$\frac{dl}{dt} = \frac{V^{-2/3}}{3V_m^{1/3}}\dot{v}\,\frac{V^{2/3}\,{}^{[E]}\!/_{[E_m]} - V^{2/3}\left({}^{V_h}\!/_{V_m}\right)^{1/3} - {}^{V}\!/_{V_m^{1/3}}}{{}^{[E]}\!/_{[E_m]} + g}$$ and using (a), (c) and (f), we obtain

$$\begin{cases} \dfrac{dl}{dt} = \dfrac{K_M}{3}\dfrac{(e - l - l_h)}{1 + {}^{e}\!/_{g}} \\ \dfrac{de}{dt} = \dot{K}_M\ g\,\dfrac{(f - e)}{l} \end{cases}$$

The scaled power for the choose way are:

Assimilation power

$$\dot{p}_A = \frac{\{P_{Am}\}}{\{J_{Xm}\}}V^{2/3}J_X = f\ l^2\ \{P_{Am}\}V_m^{2/3} = f\ l^2\ \dot{P}_{Am}$$

$$\underline{\rho_A = \frac{p_A}{P_{Am}} = f\ l^2}$$

Catabolic power

$$p_C = [E]\left(\upsilon\, V^{2/3} - \frac{dV}{dt}\right) = [E]\left(\upsilon\, V^{2/3} - \frac{\kappa\, \dot{p}_C - \dot{p}_M - \dot{p}_T}{[E_G]}\right)$$, and using (a), (d) and (e)

$$p_C\left(g + {}^{e}\!/_{g}\right) = \frac{e}{g}\{\dot{p}_{Am}\}\left(g\, V^{2/3} + l\, V^{2/3} + l_h\, V^{2/3}\right)$$

$$\underline{\rho_C = \frac{p_C}{\dot{p}_{Am}} = \frac{e\, l^2}{g + e}\left(g + l + l_h\right)}$$

Somatic maintenance

$$\dot{p}_M = [\dot{p}_M]V = \kappa\{\dot{p}_{Am}\}\frac{V}{V_m^{1/3}}$$

$$\underline{\rho_M = \frac{\dot{p}_M}{\dot{p}_{Am}} = \kappa\, l^3}$$

Endothermic heating

$\dot{p}_T = \{\dot{p}_T\}V^{2/3}$, and using (d)

$$\rho_T = \frac{\dot{p}_T}{\dot{p}_{Am}} = \kappa\ l^2\ l_h$$

Maturity maintenance

$$\dot{p}_J = V[\dot{p}_M]\frac{1-\kappa}{\kappa}$$

$$\rho_J = \frac{\dot{p}_J}{\dot{p}_{Am}} = (1-\kappa)\ l^3$$

Somatic growth

From $\kappa\ \rho_C = \rho_M + \rho_T + \rho_G$, ρ_G is defined by difference.

Maturity Growth and Reproduction

From $(1-\kappa)\rho_C - \rho_J$ for different stages.

V. Final Words

Once that we build a decision tree, it might be necessary to develop a software code generator. This software will travel the decision tree based in the physiological characteristics of the chosen species. When the software reach a box in the lower level, it will have all the information to generate a code source for the DEB model.
In this essay we only show a partial view of the decision tree. Further works might develop the "*traveler-tree-software*", "*Code-Generator-Software*" and test the resulting model against data from field study.
The "*traveler-tree-software*" begins at the highest level (the root) of the tree and it will ask to the user data of the species under study (Endo/Ectotherm, etc) traveling throughout the tree depending on the answers.
The "*Code-Generator-Software*" is activated when the "*traveler-tree-software*" reaches a lowest level in the tree. The "*Code-Generator-Software*" only might join all the equation corresponding to the path in the tree.